# Holographic Bound on Information in Inflationary Perturbations

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The formation of frozen classical perturbations from vacuum quantum fluctuations during inflation is described as a unitary quantum process with apparent "decoherence" caused by the expanding spacetime. It is argued that the maximum observable information content per comoving volume in classical modes is subject to the covariant entropy bound at the time those modes decohere, leading to a new quantitative bound on the information contained in frozen field modes in phase space. This bound implies holographic correlations of large-scale cosmological perturbations that may be observable.

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## I. INTRODUCTION

The mostly widely held theory of cosmological structure is based on primordial fluctuations that originate from quantum fields during inflation. The expansion of spacetime converts virtual quanta— the zero-point vacuum fluctuations of fields— into real classical field perturbations. Inflation theory[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16] describes how the quantum state of each field mode changes character as it expands to exceed the size of the apparent horizon— from eigenstates of number (in particular, an initial vacuum state with zero particles) to eigenstates of field amplitude, in which the quantum zero-point field fluctuations are frozen as real quasi-classical observables. Recent data, especially the concordance of microwave background anisotropy[17, 18, 19] and galaxy clustering[20], confirm many detailed features of this basic picture, including the primordial origin well before recombination, a nearly scale-invariant power spectrum, and approximately Gaussian statistics. It has long been hoped that detailed study of these quantum structures, dating as they do from close to the Planck time, might reveal qualitatively new fundamental physics connected with quantum gravity (see e.g. [21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31]).

A radical but concretely formulated conjecture about such physics, based on considerations such as quantum unitarity during black hole evaporation, on analysis of certain systems such as extremal black holes, and on the AdS/CFT duality, is that nature imposes a holographic bound on the total entropy of systems[32, 33, 34, 35, 36]. According to this conjecture, the maximum entropy of a compact system is much less than in standard field theory.

The detailed pattern of cosmic perturbations preserves more than just the power spectrum: on the largest scales, it directly records the detailed spatial configuration of the original field fluctuations frozen in during inflation. The process of freezing out creates spatially localized information from quantum states close to the Planck time that survives to the present. It is shown here that the entropy bound on fields during inflation limits the amount of information eventually carried by the final classical perturbations. Although this effect leaves the predicted mean power spectrum and Gaussian amplitude distribution unchanged, it implies a major qualitative difference from standard inflation, whose random phases and continuous spectrum contain in principle an infinite amount of information. A quantitative upper bound is derived here on the mean density of observable information in classical perturbation modes. This feature implies new correlations among modes not predicted in standard field theory. The effect can in principle provide a direct observational test of the holographic conjecture and a probe of how it is implemented in nature.

## II. FREEZING OF INFLATIONARY QUANTA

It is useful to recall the relationship of inflationary quantum field states with classically observed mode amplitudes (see e.g. [9, 10, 11, 12, 13, 14, 15, 16]).

The final eigenstates correspond to definite values of field amplitude $u$. A mode that starts off in the vacuum eigenstate at early times ends up as a superposition of these field-amplitude eigenstates at late times, with a Gaussian distribution of coefficients.

We do not observe this superposition, but only one of the amplitude eigenstates. (More accurately, we observe the late-time effects of a spacetime metric perturbation coherently imprinted by the field amplitude in one of these states). The von Neumann description of quantum measurement says that the wavefunction collapses into an eigenstate when it becomes classical. The more modern view is that the whole linearly-evolving wavefunction never collapses; however, decoherence causes the entire macroscopic world to correlate with only one of the eigenstate outcomes in such a way that the other branches of the wavefunction are unobservable.[37, 38] For a given inflationary mode of comoving wavenumber $k$, this apparent decoherence occurs near the time $t_k$ when $k = aH$, where $a(t)$ is the cosmic scale factor and $H$ denotes the expansion rate during inflation. The freezing does not depend on observations, but is a natural process during inflation as a mode's wavelength expands beyond the apparent horizon.

Any observation yields one of the eigenvalues $u$ (with probability given by the standard Gaussian amplitude

distribution.) The different $u$ are macroscopically distinguishable options like Schrödinger's live and dead cats, on a grand scale: they correspond eventually to entirely different distributions of galaxies. The information corresponding to the superposition of eigenstates is contained in observable correlations. The information content of large scale classical observables associated with modes on scale $k$ is subject to the bound on total information content at the time $t_k$.

The bound derived here is based on the assumption that although the entropy of the universe today is vastly greater than that of the same comoving volume during inflation, the "frozen" quantum fluctuations on large scales, and the metric perturbations that arise from them, are subject to the holographic entropy bounds at the time they freeze out.

## III. HOLOGRAPHIC BOUND ON FROZEN INFORMATION

It has been conjectured that nature imposes a fundamental limit on total entropy that applies to all fields. A general formulation of this limit, to which no exceptions have been found, is the "covariant entropy bound"[36]. Consider a closed spacelike 2-surface. Construct a null 3-volume $\mathcal{V}$ by propagating null rays inwards from the surface, into the future and the past, such that the areas of the inward-propagating light fronts are everywhere decreasing. The covariant entropy bound states that the entropy of $\mathcal{V}$ on either the future or past surfaces does not exceed one quarter of the area of the bounding 2-surface in Planck units.

In the inflationary context, the largest 2-sphere allowing this construction has radius just slightly less than the apparent horizon, which has an area $4\pi H^{-2}$. The corresponding bound on entropy is $S_{\mathcal{V}} < \pi m_P^2 H^{-2}$, where $m_P$ is the Planck mass. The spacelike 3-volume enclosed by this surface, on the same spacelike hypersurface used to describe the inflationary modes, has a proper 3-volume $V_H = (4\pi/3)H^{-3}$. Since it is also entirely enclosed by the null surface $\mathcal{V}$, its entropy is also bounded by $S(V_H) < \pi m_P^2 H^{-2}$. In larger 3-volumes $V > V_H$, the covariant-bound construction cannot be applied (since the inward directed light sheets have increasing surface areas); therefore, we adopt the conservative assumption that entropy on larger scales is as usual an extensive quantity proportional to $V$, bounded by $S(V) < (V/V_H)\pi m_P^2 H^{-2}$.

No matter how physically large a comoving volume eventually becomes, the frozen information contained in large-scale correlations observable at late times— that is, all the information accessible to classical observers, including information in any measurable quantities such as $u$ and $\vec{k}$— must originate within the bounded volume $V_H$. The information per comoving volume observed in modes of any scale cannot exceed the entropy bound per comoving volume corresponding to the time when they decohere. This constraint sets an upper bound on the information in all low-$k$ modes that have frozen out, and ultimately on the density of information in classical modes.

For large spatial volumes, classical entropy, defined as the logarithm of the number of states of a statistically uniform system, is proportional to 3-volume in $\vec{x}$ space times 3- volume in $\vec{k}$ space, and is independent of $a$, i.e. it is conserved by the expansion. (The 3-density of independent modes in $\vec{k}$ however increases as $\vec{x}$ volume at a given time.) Let $\mathcal{I}(k')$ denote the mean frozen information dimensionless density, per space volume times wavenumber volume, at time $t_{k'}$, in inflaton modes with $k < k'$. To respect the entropy bound per 3-volume $V$,

$$\mathcal{I}(k')V(k')[V/a^3] = (V/V_H)\mathcal{F}\pi m_P^2 H^{-2}, \tag{1}$$

where $V(k') = 4\pi a^3 H^3/3$, and $\mathcal{F} < 1$ denotes the fraction of the covariant bound on entropy carried by the information in the frozen field modes with $k < k'$ at time $t_{k'}$. The information in the frozen mode correlations does not change after $t_{k'}$, so the dimensionless information density (per space volume times wavenumber volume) in classical perturbations at late times is

$$\mathcal{I} = (9/16\pi)\mathcal{F} m_P^2 H^{-2}. \tag{2}$$

Equation (2) is the main result of this paper. It expresses the mean density in phase space of classical information, in terms of parameters $H$ and $\mathcal{F}$ characterizing inflation and quantum gravity, frozen into the metric during an approximately scale-free period of inflation.

It is worth commenting that this quantity is not the same as some other measures of information in the fluctuations, such as coarse-grained entropy[16, 31]. The dimensionless number $\mathcal{I}$ refers to a truncated Hilbert space dimension; it represents the logarithm of the number of possible different frozen field configurations, per wavenumber volume times spatial volume, representing all the different possible classical configurations of the final spacetime metric. Thus if the covariant entropy bound applies to fields during inflation, $e^{\mathcal{I}\mathcal{K}}$ represents the number of possible classical observational outcomes in a phase space volume $\mathcal{K}$.

## IV. PHYSICAL INTERPRETATION AND OBSERVABILITY

The holographic information bound implies a finite bound on the number of possible observable values for the amplitudes and wavevectors of the classical perturbation modes. The density of all of these observables taken together cannot exceed the information bound. This places a new constraint on the kinds of classical distributions that can be realized. It differs from the field-theory prediction that the amplitudes and phases of classical modes are continuous random variables [39, 40].

The observability of the holographic correlations depends on the numerical value of $\mathcal{I}$. In the general case, if

$\mathcal{I} >> 1$, it will be difficult to design realistic experiments capable of detecting this discreteness by searching for generic nongaussian features of the fluctuations. Since $m_P^2/H^2$ is at least of the order of $10^6$, and possibly much larger than that, the effect may never be observable.

On the most optimistic scenario, if $\mathcal{F}$ is small enough that $\mathcal{I}$ is of order unity (that is, $\mathcal{F} \approx H^2/m_P^2$), holographic correlations on the current Hubble scale might explain the statistical anomalies already observed in the large angle anisotropy data[41, 42]. If these oddities indeed reflect the holographic information bound (rather than simple chance or, say, a customized inflation scenario, spectral discreteness due to nontrivial topology, or other new physics[43, 44, 45, 46, 47, 48, 49, 50]), then similar correlations are predicted also to appear on smaller scales.

In some models, the information limit may also become manifest even for $\mathcal{F} \approx 1$. Consider a simple model where information is encoded in a universal, fundamental scale-invariant spectrum, with $\mathcal{I}$ discrete modes in each $\vec{k}$-space volume having a radius spanning a factor of $\log |k|$. Future experiments of various kinds have enough dynamic range to detect a discrete spectrum in this situation even if $\mathcal{I} >> 1$ [42]. For example, a complete galaxy catalog on the Hubble scale, within the capability of survey instruments such as the Large Synoptic Survey Telescope (LSST), will allow independent measurement of about $10^6$ independent spatial "pixels" (or about the same number of independent linear plane wave modes), preserving the initial phase and amplitude information from inflation, and reaching $\mathcal{I} \approx 10^6$. An advanced successor to the Laser Interferometer Space Antenna (LISA) may eventually reach the sensitivity needed to detect inflationary gravitational waves directly at 1 Hz; its frequency resolution will reach $\mathcal{I} \approx 10^8$.

Concrete, realistic predictions for such experiments require a definite holographic model of interaction of spacetime quanta with inflaton or graviton modes. In some models, the possibility in principle of discovering the nature of the holographic pixelation, or of setting constraints on the new physics embedded in $\mathcal{F}$, motivates surveys allowing detailed analysis of spatial and temporal patterns with high sensitivity and dynamic range.

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